RE-ANALYSIS OF WILLIAM FROUDE'S STUDIES OF PLANING CRAFT

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SUMMARY

One of William Froude's lesser-known contributions to the field of Naval Architecture was in conducting some of the earliest studies on planing craft. Froude's research was prompted by the idea of an inventor, Reverend C.M. Ramus who in 1872 proposed a high-speed ship concept using a flat-bottomed, stepped planing hull. Ramus later suggested the use of this hullform for rocket-propelled rams. Froude conducted towing tests on a model of the Ramus hull, as well as on a model three-point hydroplane concept of his own design. He also derived a solution for the optimum trim angle and minimum resistance of planing craft, using his recently developed formulae for estimating frictional resistance on flat plates and lift forces on submerged plates. Froude's study demonstrated that Ramus's ideas were not feasible at the time and the study received very little attention. It was not until the advent of lightweight internal combustion engines, thirty years later, that planing hulls became a reality. In the current paper, the Ramus tests are re-analysed and the data are put into a modern format. Comparisons are provided between the Ramus hull, Series 62 and a planing flat plate. The Ramus concept is shown to have had significantly more drag than would be expected from a planing hull. New model tests explore the effects of the rounded stern of the Ramus hull, and show that this feature significantly increases the resistance. Froude's derivation of the minimum resistance of planing craft is discussed and contrasted with modern methods for prismatic planing hull resistance prediction.

NOMENCLATURE

A_M	Midship Area (m^2)
A_{WP}	Waterplane Area (m^2)
A_P	Projected plan area of planing bottom measured to chines (see Clement and Blount, 1963) (m^2)
$A_P/\nabla^{2/3}$	Bottom Loading Coefficient
B	Beam of Planing Surface (m)
B_{PX}	Maximum Chine Beam of Planing Surface (see Clement and Blount, 1963) (m)
C_B	Block Coefficient ∇/LBT
C_M	Midship Section Coefficient A_M/BT
C_{WP}	Waterplane Area Coefficient A_{WP}/LB
C_Δ	Beam Loading Coefficient ∇/B^3
C_V	Speed Coefficient (Beam Froude Number) $V/\sqrt{gB}$
Fn	Length Froude number $V/\sqrt{gL_{WL}}$
F_∇	Volumetric Froude number $V/\sqrt{g\nabla^{1/3}}$
H_{STEP}	Step Height (m)
LOA	Length Overall (m)
LCG	Longitudinal Centre of Gravity, measured forward of transom (m)
L_M	Mean wetted length (m)
L_P	Length of planing surface measured from transom to chine intersection with centreline (see Clement and Blount, 1963) (m^2)
L_{WL}	Static Waterline Length (m)
N	Normal force on bottom of planing hull (N)
T	Tangential force on bottom of planing hull (N)
T_A	Draft aft above baseline (m)
T_F	Draft forward above baseline (m)
R	Total Resistance (N)
Re	Reynolds Number $= VL_M/\nu$
$R_{T,M}$	Total Resistance of Model (N)
R_R	Residuary Resistance (N)
S	Wetted surface area (m$^{2)}$
V	Vessel Speed (m/s)
W	Weight (N)
Δ	Displacement (kg, t)
∇	Volume (m^3)
β	Deadrise (deg)
ρ	Density of water (kg/m^3)
τ	Hydrodynamic Trim Angle (degrees)
τ_{RAD}	Hydrodynamic Trim Angle (radians)
υ	Kinematic viscosity (m^2/s)

1. INTRODUCTION

In the early 1870's an English reverend, C.M. Ramus, believed that war at sea could be rendered obsolete by the introduction of a new weapon of unlimited power that would "sweep away all existing navies." This weapon (Figure 1) was the "Rocket Ram" a proposed 35 m, 120 t rocket-propelled weapon that could skim across the water's surface a distance of two miles and impact its target near the waterline at 200 m/s, ramming with eight times the energy of a 10000 t warship at 15 knots. The extreme speed of this weapon, far beyond the limits of conventional ships, necessitated the development of an entirely new type of hull that would be bodily lifted at speed and traverse over the surface like a "flat stone skimming along the water", "virtually obliterating water resistance." [1]

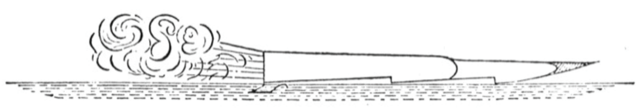


Figure 1: Sketch of Rocket-Ram by C.M. Ramus [1]

The rocket ram proposal was one of many high-speed ship concepts proposed by Ramus to the Admiralty. The Admiralty tasked William Froude with investigating Ramus's claims using his newly-built towing tank and model-ship extrapolation procedure. Froude concluded that the concept of a high-speed ship that skims across the surface of the water was theoretically interesting, but very impractical due to the limited power available, and because of the dangers involved in operating such a ship in a seaway. At the time, it was inconceivable that, within half a century, there would be thousands of planing craft powered by lightweight internal combustion engines.

Froude conducted an extensive study to evaluate the Ramus proposal. Froude's study included high-speed towing tests on a variety of models, including stepped planing hulls and three-point hydroplanes. Additionally, Froude developed theoretical relations for the minimum resistance and trim of planing craft, using an empirically-derived planing lift equation. These studies were not useful at the time because the conclusion was that Ramus's ideas were full of misconception and error. However, when looked at from a modern perspective, we realize that Froude's work was a significant scientific accomplishment that has been mostly forgotten. In the present paper, Froude's model tests and his analysis of minimum resistance of planing craft are reviewed in the light of our current (and still developing) knowledge in this area.

Remarkably, 140 years after Froude initiated this study, high-speed planing craft, and especially stepped hulls, are still an active area of scientific research. Morabito and Pavkov [2] and Lee and Pavkov [3] measured the resistance of single- and two-step, low-deadrise planing hull models. Matveev [4] produced a theoretical solution for the lift of two-dimensional flat plates planing in tandem. White and Beaver [5] studied single-step hulls experimentally. Garland and Maki [6] studied two-dimensional single step hulls numerically. Savitsky and Morabito [7] developed a method of estimating the equilibrium conditions of single-step planing hulls. In the light of all of these emerging studies, it seems appropriate that the pioneering work of William Froude be explored.

2. PROPOSAL BY RAMUS

In 1872, the Reverend C.M. Ramus wrote a letter to the Lords Commissioners of the Admiralty suggesting a new form of ship consisting of two wedges positioned fore and aft. The purpose of the invention was to cause the ship to generate dynamic lift at high speeds, reducing its resistance (Figure 2). Unlike hulls previously proposed by other inventors (as in Figure 3), the two planing surfaces would be placed so that the ship would always maintain proper trim. Ramus's early experiments indicated that such a form might allow for the speed of steamships to be doubled. Three advantages were cited:

1. Resistance would increase very little with speed
2. Increasing beam would not increase resistance, but would provide stability and reduced draft
3. Wind resistance would be the dominant component at high speeds, suggesting that speeds of 25 to 45 knots might be possible.

The correspondence between Ramus and Froude is summarized in an 1875 article in *Naval Science* [1]. It is also included in a more complete form in the House of Commons Hansard of 31 March 1873 "Correspondence between the Admiralty and the Reverend C.M. Ramus on certain Experiments conducted by them." [8]

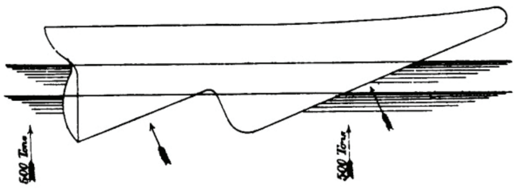


Figure 2: Stepped hull proposed by Ramus [1]

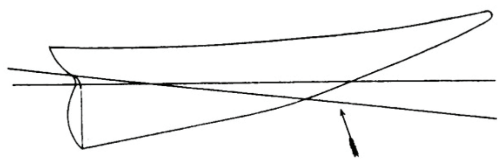


Figure 3: High Speed Ship Repeatedly Proposed by Early Inventors [1]

3. FROUDE'S MODEL TESTS ON RAMUS HULL

Upon receiving Ramus's letter, Froude wrote back to determine the slopes that Ramus proposed, as the initial sketch (shown in Figure 2) appeared to be too high. Ramus clarified that an angle of 2 to 3 degrees with respect to the water surface would be preferable. Figure 4 shows the revised profile view of the hull. Table 1 shows the list of principal characteristics of the Ramus hull. The final ship had a slope of 1:50 on the planing surfaces.

Froude had two models constructed of the Ramus design, one at 1:36 scale and one at 1:108 scale. The principal characteristics of the models are shown in Table 2.

Froude expanded the results of the two models to the same scale to demonstrate that the scaling method employed was satisfactory. The method that he used is very similar to our current method for planing craft, with two exceptions. (1) It had not been discovered yet that the friction coefficient depended on Reynolds number. Froude estimated the frictional resistance of the model using friction factors from a plate of similar length and material as the model. (2) The report makes no mention of measurements of dynamic wetted surface area, which is typically used for expanding planing hull resistance today. The use of the larger static wetted area

(typical for displacement ship expansions) results in inaccurate full-scale expansions for planing craft.

To make the tests results more accessible, the author has digitized the plots of the data that were found in the Parliamentary record [8] and normalized them using non-dimensional coefficients more relevant to modern designers, such as volumetric Froude number.

Table 1: Principal Characteristics of Ramus Hull

LOA	109.7 m
Beam	15.25 m
Draft	2.13 m
H_{STEP}	~0.9 m (estimated from sketch)
Δ	2540 t
L_{WL}	~103.6 m (estimated from sketch)
C_M	1.0
C_{WP}	1.0
C_B	0.73
$L_{WL}/\nabla^{1/3}$	7.6
$L_P/\nabla^{1/3}$	8.10 ($L_P \approx L_{OA}$)
L_{WL}/B	6.8
H_{STEP}/B	6%
$A_P/\nabla^{2/3}$	9.13 ($A_P \approx L_{OA}B$)
C_Δ	0.70

Table 2: Principal Characteristics of Two Models Tested

Scale	1:36	1:108
LWL	2.87 m	0.959 m
Displacement	53.1 kg (FW)	1.97 kg (FW)

Figure 4: Improved Hull Proposed by Ramus [1]

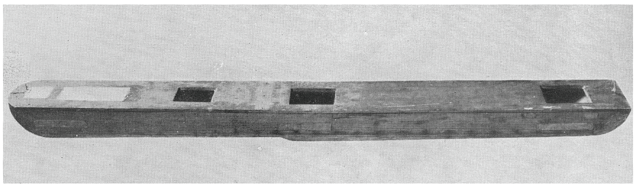


Figure 5: Stepped planing hull proposed by Ramus and tested by Froude [9]

3.1 TRIM AND HEAVE

Froude recorded trim and bodily rise and fall of the model as a function of speed for the two models by the use of trim gauges located fore and aft. Figures 6 and 7 show CG rise and trim angle (degrees) as a function of volumetric Froude number. Typically, the expected trend would be an increase in trim up to a volumetric Froude number somewhere near 3 and then a steady decrease in trim (see plots from Clement and Blount, [15] for instance). Unfortunately, Froude did not present the trim data for the higher speeds and so it is not known whether this drop was seen.

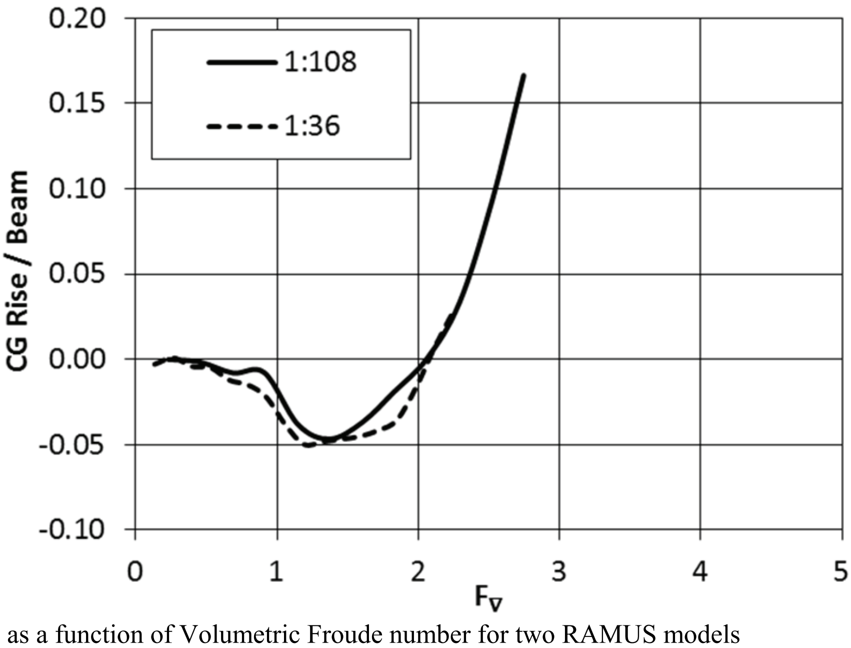


Figure 6: CG Rise as a function of Volumetric Froude number for two RAMUS models

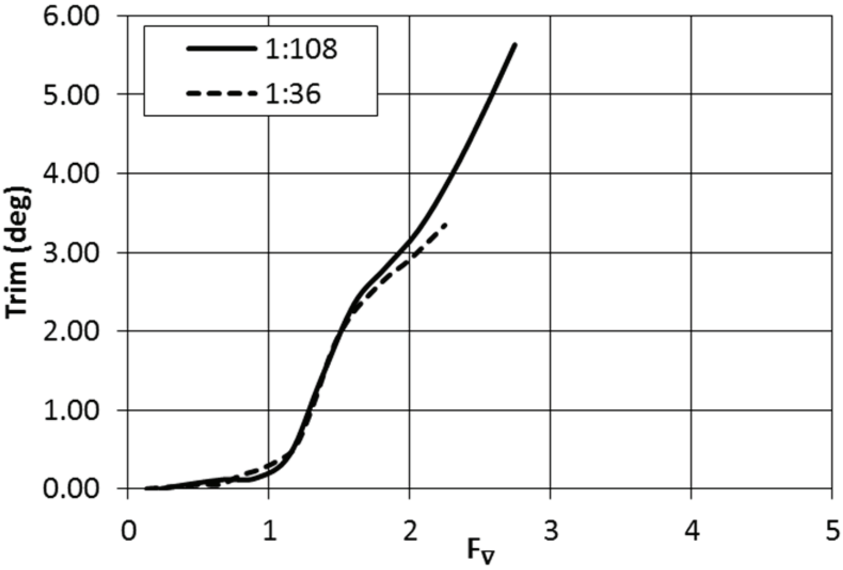


Figure 7: Baseline trim as a function of Volumetric Froude number for two Ramus models. (hydrodynamic trim of planing surfaces is baseline trim + 1.15 degrees)

3.2 WETTED AREA AND RESIDUARY RESISTANCE

In order to properly expand planing-hull test data, it is necessary to separate the frictional and residuary components of resistance. This is only possible if the dynamic wetted-surface area is known from the tests. Use of static wetted-surface area over-predicts the frictional component of resistance. Since the friction coefficient at full-scale is smaller than that at model scale, use of a wetted area that is too large results in an under-prediction of the full-scale resistance.

Today, wetted-surface area is often measured using under-water photographs of the planing hull; however, in their absence it is necessary to estimate wetted area based on the recorded measurements of the change in draft, and check it against sketches that Froude provided for certain speeds (Figure 8). The wetted area of the model was estimated from the measurements of the change in draft as follows:

$$L_M \approx L_{OA} T_A / (T_A - T_F)$$
$$L_M \leq L_{WL}$$

where L_M is the mean wetted length ($S = L_M B$), and T_A and T_F are the drafts forward and aft to the baseline. T_F becomes negative when the bow comes out of the water. This simplified equation does not include the additional corrections for the 1:50 slope of the planing surfaces with respect to the baseline, or the wave rise that occurs on flat-bottomed hulls. These corrections are relatively small and they offset each other.

Figure 9 shows the mean wetted length estimated using the previous equation as a function of volumetric Froude number. Froude did not report trim data above a volumetric Froude number of 3 and therefore the remainder of the curve had to be estimated. This estimate was done assuming that the hull was planing on two steps with a ventilated cavity behind the forward step, which should be the case for such a large step height. The estimates follow the trend of the curves provided by Morabito and Pavkov [2] for the wetted area of stepped planing hulls.

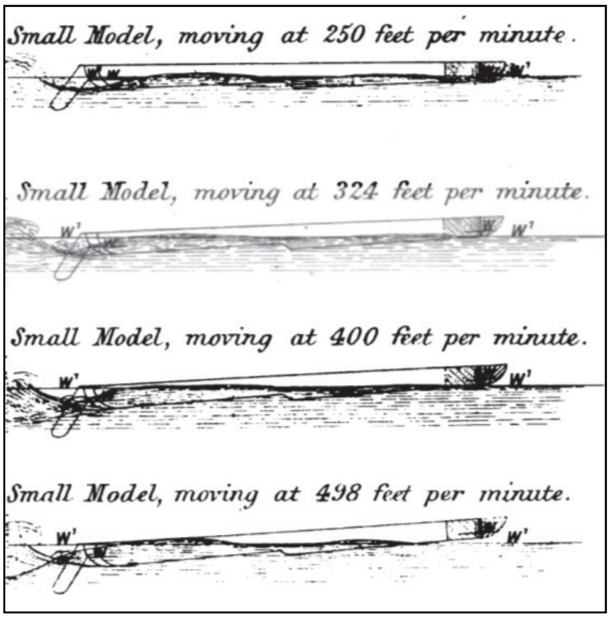


Figure 8: Sketches of Wave Profile on Model at Various Speeds [8] $F_\nabla \approx 1.1,\ 1.5,\ 1.8,\ 2.3$

A sensitivity analysis was performed to determine the effects of errors in wetted area on the expanded resistance-to-weight ratio. An error in wetted length of 10% of the static waterline length results in a 6% difference in the resistance-to-weight ratio at full-scale.

The frictional resistance of the model was estimated by assuming a wetted surface area equal to $S = L_M B$, which assumes no side-wetting. The ITTC 1957 line was used to determine the friction coefficient, using mean wetted length as a basis for Reynolds number. It was assumed that the tests were conducted in 20°C fresh water, which is typical for towing tanks. Figure 10 shows the residuary-resistance to weight ratio estimated using this method. One of the more noticeable features of Figure 10 is the large difference between the results of the 1:108 and 1:36 models. This is also evident in Figure 7, with the small model having higher trim. The variation in trim and resistance indicates a strong scale effect at the higher speeds, not ordinarily seen in planing model tests.

One plausible explanation for these problems is that the transom is somewhat rounded (see figures 4 and 5). It is probable that the flow at the transom (and possibly the chines too) does not separate cleanly, causing a low pressure region near the stern and possibly side wetting. This low-pressure region results in an increased trim angle and therefore an increased residuary resistance. Further indication of this problem is seen in Froude's observation of a trim instability. These types of flows should not be expected to scale well between model sizes because of the uncertainty in the separation point.

The differences seen in Figure 10 are unusual, as small planing-boat models typically scale well. The flow breaks cleanly from the sharp chines and transom, minimizing the most problematic scaling issue of ship models, viscous pressure resistance. Viscous pressure resistance often forms as a result of a large boundary layer or separated flow at the stern of a ship, and is highly dependent on scale and Reynolds number.

The author reviewed some of the relevant literature on the subject to determine whether the differences seen between the small and large models should be expected. Tanaka [10] presented the results of cooperative resistance tests on geometrically similar models of a high-speed semi-displacement craft. He found that below 0.75 metre model length there were scale effects in the trim angle and spray. Above 1.5 metre length, the resistance did not have any serious scale effects. The smaller Ramus model was 1 metre long. In contrast to Tanaka's findings on semi-displacement craft, many of the models used to develop the prismatic planing hull lift equations (Savitsky [11] or Shuford [12]) had beams of 200mm 100mm and as low as 50 mm on one occasion [13]. In many cases, the chines and transom of these planing models were inlaid with sharp brass to ensure that the flow broke cleanly from the hull instead of clinging to the sides. These results were shown to scale well from small to large models. Based on this review of planing hull scaling, the author concludes that the 1 m length and 141 mm beam of the small Ramus model are smaller than ideal, but the scale effects are greater than would otherwise be expected.

Whereas it has been shown that the small model is of acceptable size, the large scale effects seen between the two models indicate that the flow is not cleanly separating from the rounded transom, causing large differences between models.

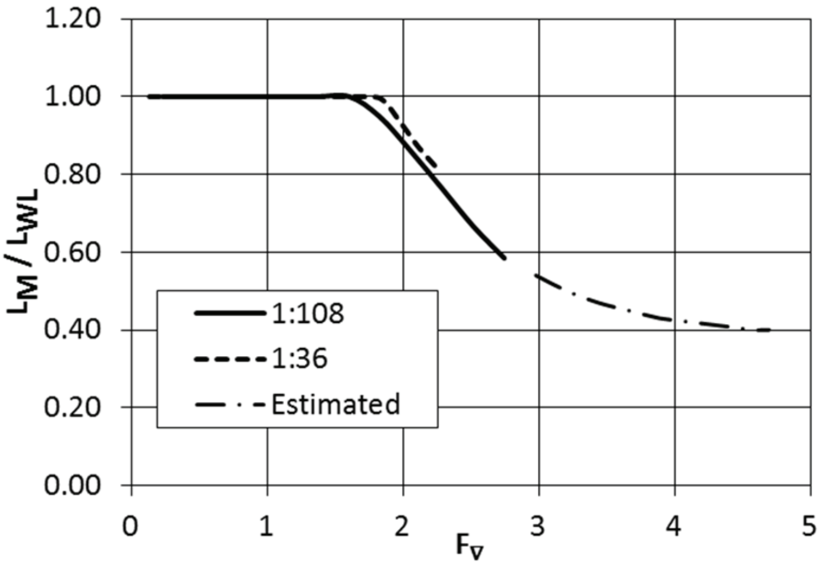


Figure 9: Mean Wetted Length as a Function of Volumetric Froude Number Calculated from Trim Data. (Note: mean wetted length is estimated at speeds above a volumetric Froude number of 3).

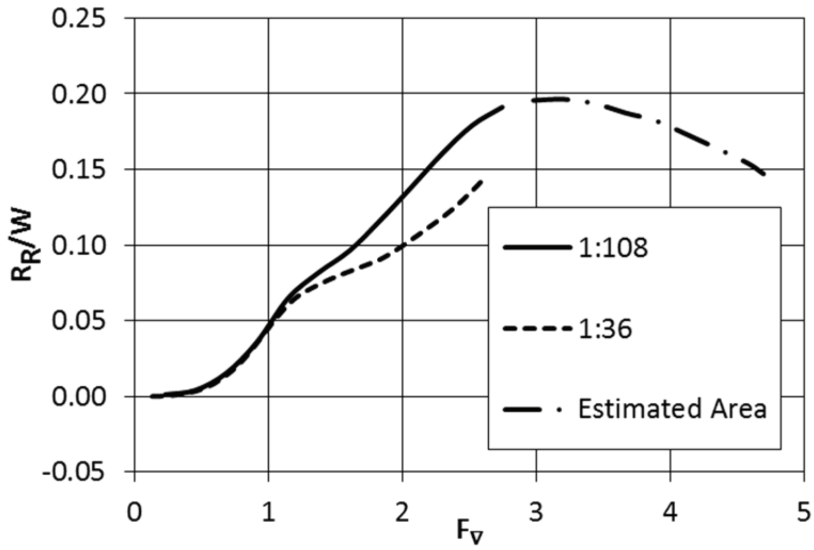


Figure 10: Residuary-resistance–to–weight-ratio for Ramus models versus Volumetric Froude number.
(Estimate line is shown at speeds at which the total resistance was measured but wetted area had to be estimated)

3.3 TRIM INSTABILITY

Froude [8] discusses an unusual observation with the smaller Ramus model. At some of the highest speeds tested, "in apparent defiance of a law of equilibrium in its simplest form," the model ran at the attitude shown in the top sketch in Figure 11, in which the longitudinal centre of gravity overhung all of the water support. Typically at some point during the run, the bow would drop and the hull would regain the typical running attitude seen in the sketch in the bottom of Figure 11. This observation was witnessed by a number of people and documented in the measurements of rise at the bow and stern. Figure 12 shows a plan view of the phenomena. The sketch clearly shows the maximum wetted length aft of the step.

Based on more-recent experience with rocker on planing hulls, the author believes that these phenomena occurred as a result of strong low pressures forming on the curved underside of the stern, a result (related to Bernoulli's equation) of the water accelerating around the curve. In order to produce the extreme case shown in Figure 11, these pressures must have been exceptionally low.

Figure 13 shows the effect of stern suction on model resistance (determined from measurements before and after the bow dropped during the run). Above a volumetric Froude number of 2.5, the instability manifests itself in two possible equilibrium conditions, the suction condition (with higher resistance) and the released condition (with lower resistance). This instability is non-oscillitory.

The acceleration of the flow around the curved portion of the stern, and the separation of the flow off this curve result in a dynamic instability (two equilibrium conditions) and also indicate the presence of strong scale effects: a larger or smaller scale model may have separation occurring at different times. This may be the reason for the discrepancies in the resistance and trim of the larger and smaller Ramus models seen in Figures 7 and 10.

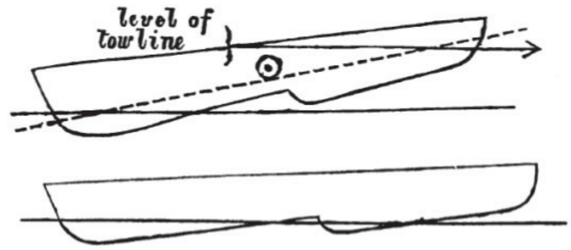


Figure 11: Sketch of Stern Suction Instability. Top, Unnatural Running Attitude; Bottom, Typical Running Attitude [8]

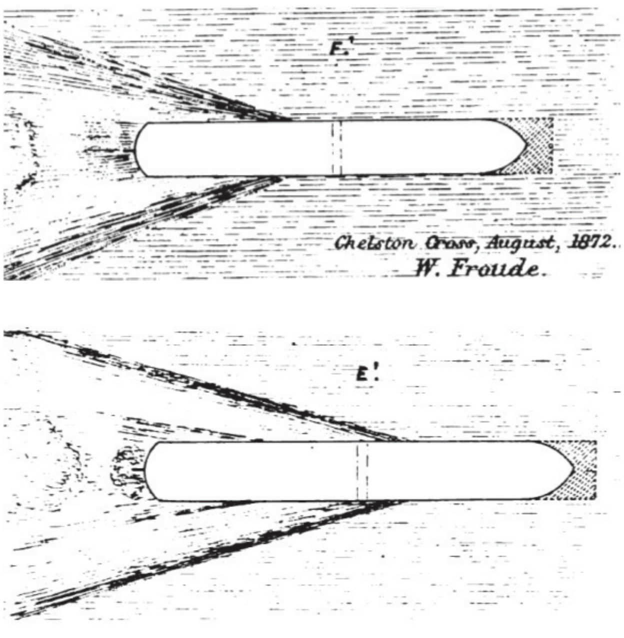


Figure 12: Plan view of Model with and without "dead water" hanging onto the stern. Top, Unnatural Running Attitude; Bottom, Typical Running Attitude. [8]

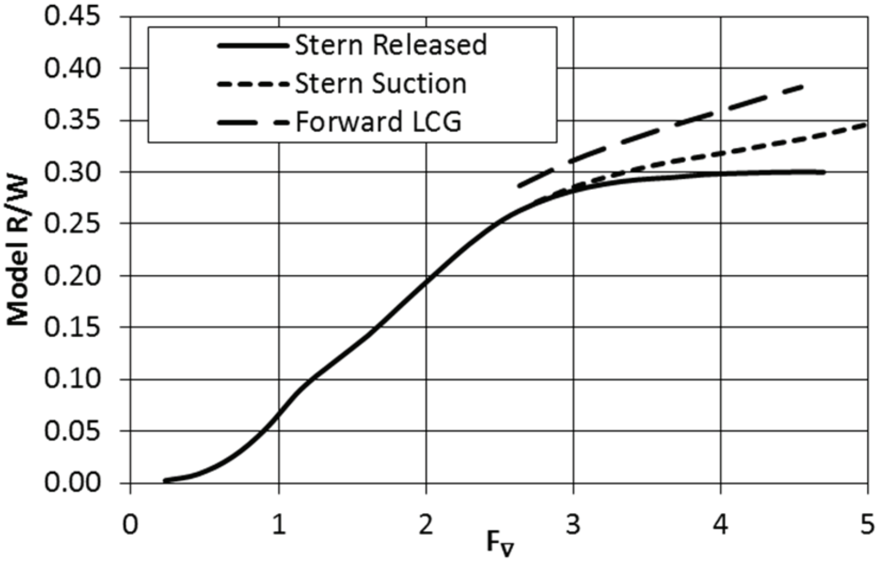


Figure 13: Total Resistance-to-weight ratio of Small RAMUS model, showing effect of suction at stern and LCG variation

3.4 LCG VARIATION TESTS

Figure 13 includes the results of a study of longitudinal centre of gravity (LCG) variation, in which the LCG was shifted 185 mm (approximately 20% of the waterline length) forward on the small model. Froude does not discuss this study; however, it is evident that the bow-down trim increases resistance. This large bow-down moment likely resulted in a sub-optimum running trim.

3.5 RAMUS COMPARED TO OTHER HULL FORMS

Froude compared the results of the model tests of the Ramus hull with two other models: (1) a scaled-up version of the famed *Greyhound*, for which full-scale towing tests had been performed (Froude [14] and Figure 14); and (2) a model with the same initial lines as the *Greyhound*, but with dimensions scaled to have the same length, beam and draft as the Ramus hull.

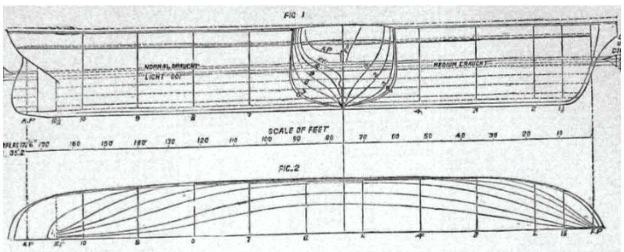


Figure 14: *HMS Greyhound* [14]

These comparisons are somewhat limited in applicability, because the maximum volumetric Froude numbers that the two *Greyhound* variants were tested at were much lower than where a stepped planing hull would be applicable. This is because the original Ramus ship concept was proposed to be over 100 m long, so even speeds of 50 knots would be in the low semi-displacement speed regime.

The author felt that the comparisons would be more illustrative if the Ramus hull was compared to familiar planing hull forms, such as the Series 62 (Clement and Blount [15]) or a simple flat plate (Savitsky [11]). It is acknowledged that changes in LCG greatly affect the performance of planing craft and, therefore it is somewhat difficult to make comparisons without trim optimizations; however, for the sake of illustration, the LCG for all three methods was kept near the original location (estimated to be somewhere around 45% LWL forward of the transom).

The results of the Ramus tests were expanded to a notional 100,000 lb (45,000 kg) hull to allow direct comparisons with the Series 62 total resistance-to-weight ratio plots. For the Series 62 comparison, the closest model in the series was selected (Model 4669). A flat plate was also selected using the same beam, displacement and LCG location of the 100,000 lb (45,000 kg) scaled Ramus hull. The principal characteristics of the three hulls are given in Table 3. The small Ramus hull data were used, as they had the highest Froude numbers tested. The design LCG and stern suction released condition was used (lower curve in Figure 13), as Froude recorded data prior to and after the stern suction was released.

Figure 15 shows the comparison between the Ramus hull, Series 62 and the Savitsky flat plate. At a volumetric Froude number of 2 the Ramus hull has nearly three times the resistance of a flat plate, a remarkably large difference, considering that the hull parameters are very similar. One notable difference between the Ramus hull and a flat plate is the step in the

hull. Many studies have shown that steps tend to increase resistance at pre-planing speeds, but only by around 15% [2]. The large difference in resistance is indicative of a much larger problem with the hull.

The stern suction instability shown in Figure 11 seems to be the most-likely cause of the increased resistance shown in Figure 15. In order to achieve the running attitude shown in the sketch, a large downward force would have to be generated at the stern. As the speed increased, it is likely that the flow began to properly separate, and the resistance approached that of a typical planing craft.

Table 3: Principal Characteristics for Performance Comparisons

Displacement	45,300 kg (for all)
LCG (forward transom)	12.2 m (for all)

	Ramus	**Flat Plate**	**Series 62**
B_{PX}	3.99m	3.99m	4.31m
L_P	28.7m	N/A	30.2m
L_P/B_{PX}	7.2	N/A	7.0
$A_P/\nabla^{2/3}$	9.13	N/A	8.5
β	0 deg	0 deg	15 deg

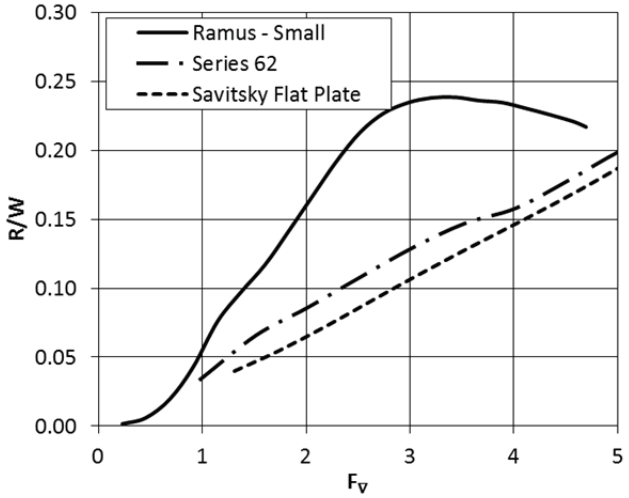


Figure 15: Resistance Comparison between Ramus hull, Series 62 and Savitsky Flat Plate. (Resistance-to-weight ratio versus volumetric Froude number for 100,000 lb (45,000 kg) vessel)

4. NEW RAMUS STERN TESTS

The resistance comparison shown in Figure 15 indicate that the Ramus hull had nearly three times the resistance of an equivalent flat plate at a volumetric Froude number of 2. Further, an unusual dynamic instability was present, in which the centre of gravity overhung the forward extent of the wetted area. The author hypothesised that these problems were the result of a large suction force as the flow accelerated over the curved portion of the transom. To test this hypothesis, a short series of model tests was conducted in the towing tank at the United States Naval Academy to investigate the effect of the rounded stern on the forces generated by the planing hull.

4.1 MODELS AND INSTRUMENTATION

Two towing models were constructed and tested. Both models were 1m long with 140 mm beam, approximately the same size as the small Ramus model. Both models included a flat bottom with no step. The step, which is present on the original Ramus model, was omitted in order for these tests to represent the effect of only one variable – the rounded stern.

The first model had the stern rounded, with an 85 mm radius in plan, 50 mm radius in profile and 10 mm radius on the chines. These radii were estimated from the sketches and photos in Figures 4, 5 and 12. The radius on the chines was not shown in Froude's figures; however, the author assumed, based on the rounding of the stern, that the chines would have been smoothed as well. The second model was left flat, with sharp edges at the chines and transom (typical of planing hull test models). Figure 16 shows a photograph of the two models. The two models were constructed from expanded PVC, painted, and sanded with 600-grit sandpaper.

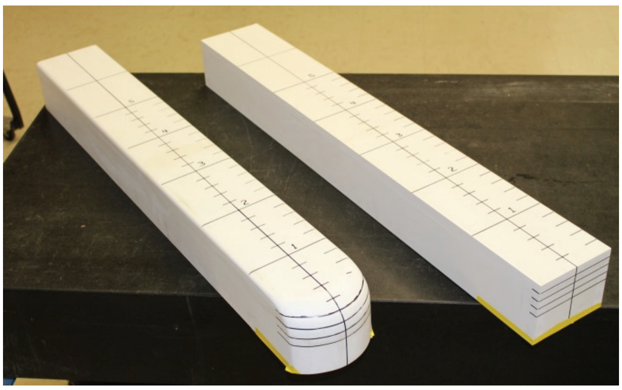


Figure 16: Photograph of the Two Models

The models were tested, fixed in heave and trim, at a running attitude similar to the stern of the Ramus model at mid-range speeds, 6-degree trim angle and static draft corresponding to a wetted length of 3 beams (when running, the wave-rise resulted in a mean wetted length of 3.4 beams). The models were tested using the same style of apparatus used in [16]. This fixed-heave testing permits measurement of forces both normal and tangential to the bottom of the planing craft.

Figure 17 shows a diagram of the orientation of the measurements of tangential force *T* and normal force *N*. The reason for measuring in this orientation (rather than the standard lift and drag) is that, at high speeds (when the flow breaks cleanly from the chines and transom), the tangential force is equal to the friction force on the planing surface. This conveniently separates the friction forces, which are a function of the Reynolds number, from the pressure forces which depend on the Froude number. Lift can be related to the normal force and tangential force as follows:

$$L = N\cos\tau - T\sin\tau$$

where *N* is the normal force, *L* is the lift force and *T* is the tangential force. The dominant component of lift is the normal force. Figure 18 shows a photograph of the test setup. The model and dynamometer can be adjusted vertically with a rack and pinion located on the towing carriage. Figure 19 shows a photograph of the towing dynamometer removed from the hull. The plate on the bottom mounts to the hull. Two gauges are mounted, one measuring force tangential to the bottom of the hull and one measuring force normal to the bottom. The angle of attack was adjusted by loosening the handles on either side and setting the angle with a digital inclinometer set on the bottom of the hull.

Although the model was set at a draft corresponding to a wetted length of 3 beams, planing hulls experience a wave-rise in which the intersection between the water and the hull is somewhat further forward than the zero-speed case. Underwater photographs were used to measure the wetted length while the model was running.

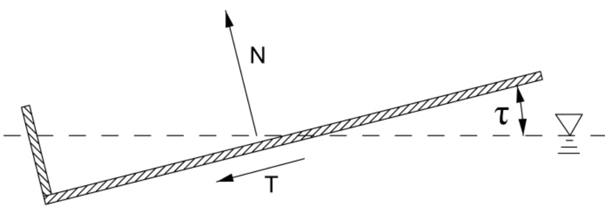


Figure 17: Orientation of Measurements

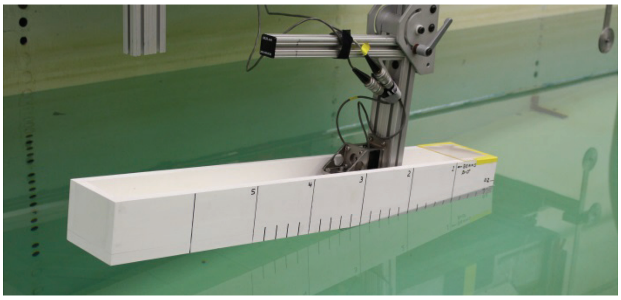


Figure 18: Photograph of Test Setup

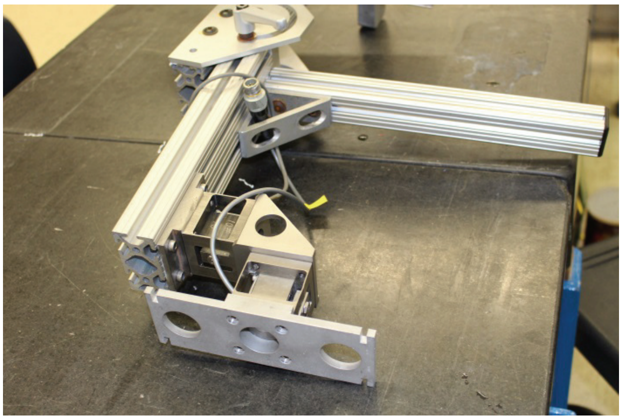
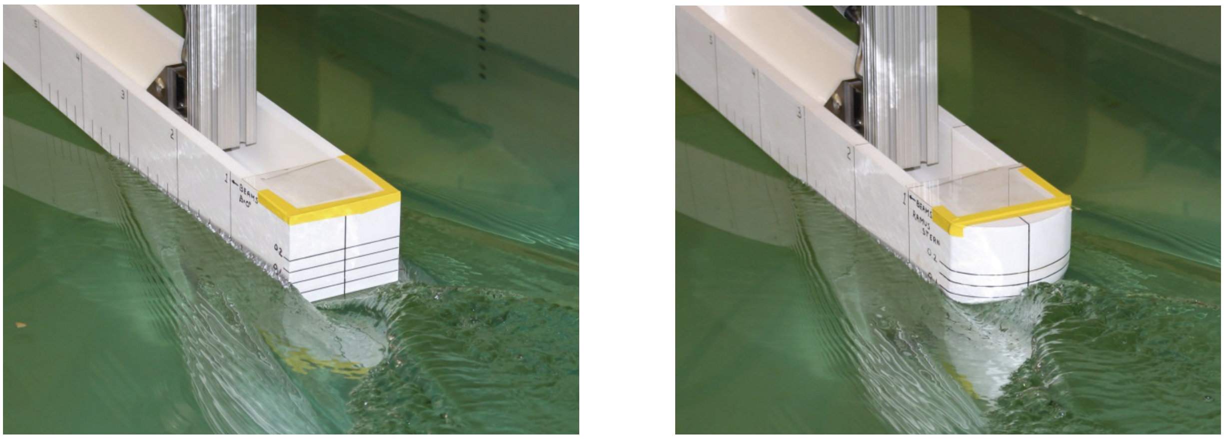


Figure 19: Dynamometer used for Testing

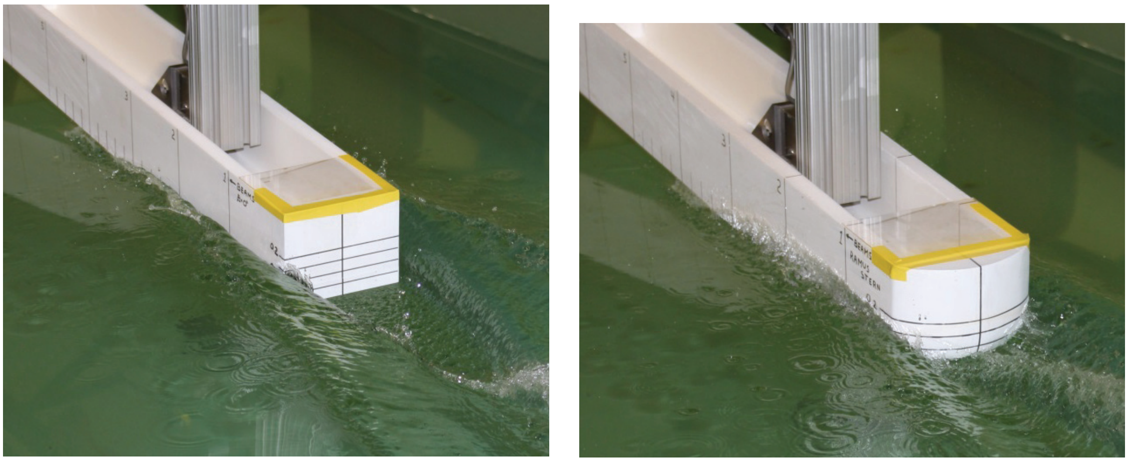
4.2 TEST RESULTS

The model tests results are tabulated in Appendix B. Figure 20 provides comparisons of the stern wetting of the two models at speed coefficients $C_V = V/\sqrt{gB}$ = 1.0 and 3.1. The figure shows that, at $C_V = 1.0$, the flow cleanly separates off the square stern, while remaining attached to the rounded stern. At $C_V = 3.1$, the highest speed tested, a noticeable trough forms aft of the square-stern model, whereas the rounded-stern model still has some amount of water clinging to it. Additionally, the flow breaks cleanly from the chines of the square model, whereas it wraps up the sides of the rounded model, resulting in additional frictional resistance.

Figure 21 shows a comparison of normal force measured between the two models. Speed coefficient is used in place of volumetric Froude number because the lift force is a dependent variable in these tests. The normal force was measured by first zeroing the instrumentation with the model in air, then lowering the model to the correct immersion. The rounded-stern model has less initial normal force due to reduced volumetric displacement. As speeds increase up to $C_V = 1$, both models see a reduction in normal force. This has previously been observed by Morabito (2013) and is the reason why planing hulls squat at pre-planing speeds. As speed further increases, the normal force of the square-stern model begins to increase until it is substantially larger than the hydrostatic normal force. In contrast, the Ramus stern continues to have more negative normal force with increasing speed. Because of the large difference in



C_V = 1.0



C_V = 3.1

Figure 20: Comparison of Stern Flows at two Speed Coefficients (Left, Square Stern; Right, Ramus Stern)

normal force seen between the square-stern model and the rounded-stern model, it is evident that the rounded stern causes a very large component of downward force, enough to cancel out the hydrodynamic lift that would ordinarily be generated on the plate. This downward force, and the bow-up pitching moment which it would cause, also explains Froude's observations of dynamic instabilities and the very large resistance shown in Figure 15.

Figure 22 shows a comparison of tangential force (which is a combination of frictional resistance and hydrostatic buoyancy acting on the transom). At zero speed this tangential force is negative (i.e. pushing on the stern) because of the hydrostatic buoyancy acting on the stern. This negative force does not imply that the hull propels itself. At zero speed the hull is in static equilibrium because the *horizontal component* of the normal force and the *horizontal component* of the tangential force are equal and opposite (this can be shown by integrating the hydrostatic pressures acting on the object):

$$N \sin\tau = -T\cos\tau \text{ (at zero speed)}$$

At higher speeds the transom ventilates on planing craft and the hydrostatic component of the tangential force is equal to zero. Therefore, at high speeds the tangential force is equal to the friction alone. The figure shows that, at low speeds, both hulls have similar tangential force. At high speeds the Ramus model has higher tangential force due to the flow clinging to the wetted sides of the hull, increasing frictional resistance.

The primary conclusions of the test are:

- The stern suction was large enough to cancel out the hydrodynamic lift that was generated by the plate at 6-degree trim.
- The rounded chines result in side wetting and increased frictional resistance.

An error analysis was conducted. The largest source of error in these tests was the draft of the model, as the

water level fluctuated by small amounts during the test. This resulted in some variability of measurements of normal force, which is highly dependent on draft. Table 4 summarizes the standard deviation of test measurements. Standard deviations of the normal force and tangential force measurements were estimated by computing the standard deviation of the static runs and comparing with a limited number forward-speed repeatability runs. These values are shown as the error bars in Figures 21 and 22.

Table 4: Standard Deviation of Test Measurements

Draft	+/- 0.7 mm
Trim	+/- 0.1 degrees
Roll	+/- 0.1 degrees
Wetted Length	+/- 0.05 beams
Speed Coeff.	+/- 0.005
Normal Force	+/- 0.7 N
Tangential Force	+/- 0.2 N

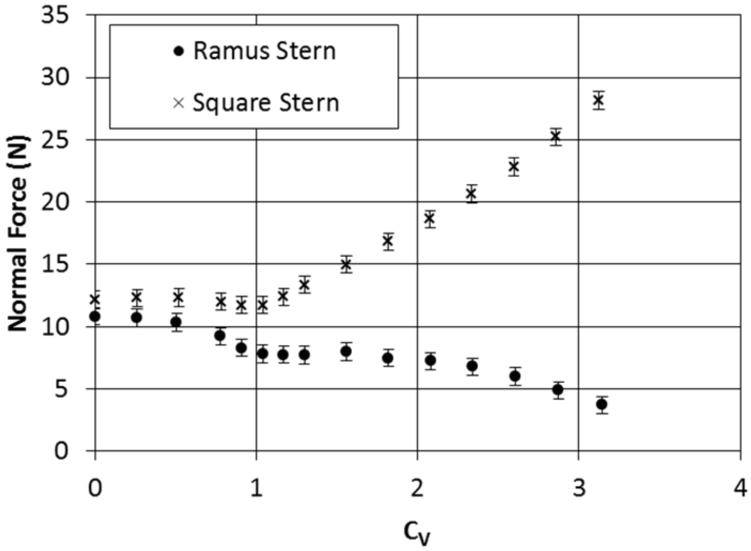


Figure 21: Comparison of Normal Force between Square and Rounded Stern Models

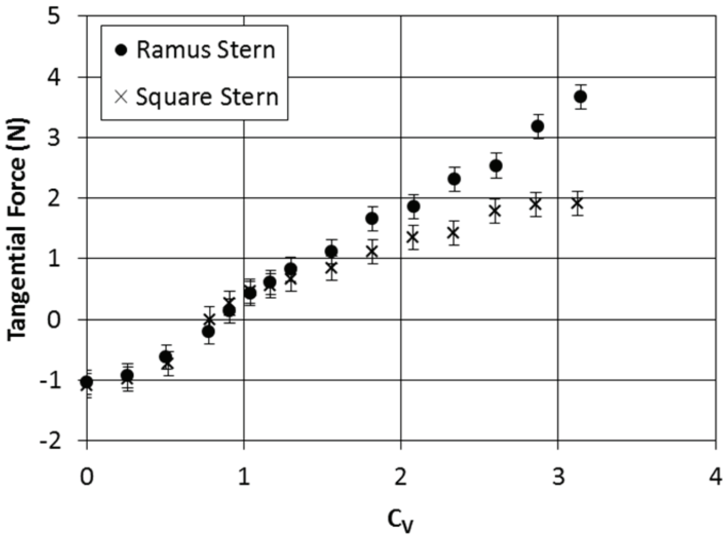


Figure 22: Comparison of Tangential Force between Square and Rounded Stern Models (negative tangential force is a result of hydrostatic buoyancy acting on the transom at zero speed)

5. FROUDE'S DERIVATION OF THE MINIMUM RESISTANCE OF PLANING CRAFT

One of Ramus's main claims was that, at high speeds, the hull would be lifted out of the water and the resistance would eventually be eliminated. This elimination of resistance was not observed in the tests of the Ramus model, even though Figure 6 shows that the hull was lifted. Froude felt that, because the aft planing surface of the Ramus design ran in the wake of the forward one, the trim was not constant, and the resistance was not minimized. As an alternative, Froude developed a design in which the rear planes were positioned in undisturbed

water, allowing the craft to maintain proper trim regardless of speed. Figure 23 shows a sketch of Froude's design of a three-point hydroplane.

The 0.5 kg model was tested up to a volumetric Froude number of 6.7. At these speeds, Froude observed that the hull had less than 3 mm immersion. Froude found that the resistance continued to increase with speed, although it reached a maximum some time before the maximum speed was reached. This indicated that there was some point at which resistance would stop increasing; however, the resistance was not eliminated. Froude tested the model with both 1:15 and 1:50 slopes (3.8 and 1.1 degrees respectively). The 1:50 slope model had around 15% more resistance and was not lifted as much. Froude did not provide the resistance data from these tests in the paper.

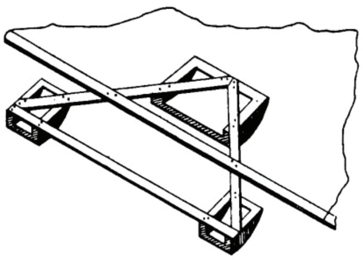


Figure 23: Froude's Three-Point Hydroplane [8]

The experiment was a clear contradiction of Ramus's claim of eliminating resistance at high speeds. Following this experiment, Froude prepared a simple derivation for the minimum resistance of planing craft showing, that as speed is increased, the resistance does not approach zero. Froude's derivation is provided in Appendix C of this paper. The analysis is repeated here in a non-dimensional format more familiar to modern planing-hull designers.

5.1 COMPONENTS OF RESISTANCE

The planing hull is represented by a flat plate running at a very high speed, in which the flow separates cleanly from the transom. The contribution due to buoyancy is negligible. The resistance of this planing craft consists of two primary components, shown in Figure 24.

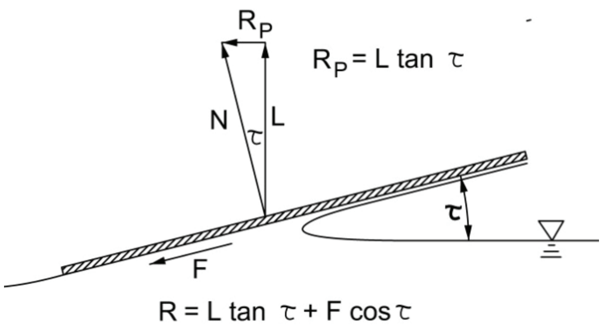


Figure 24: Resistance of a Flat Plate

First, pressure drag, which the horizontal component of the normal force acting on the bottom. In the absence of large vertical forces, such as inclined propeller thrust (which is not included in this simplification), the vertical component of the normal force acting on the bottom (lift) must be equal to the weight of the boat. Therefore,

$$R_P = L \tan \tau \approx W \tan \tau$$

where
R_P = pressure drag (N)
L = lift of planing surface (N)
W = weight the hull supports (N)
τ = angle of inclination of the planing surface with respect to the horizon (deg)

Second, the horizontal component of the frictional force on the wetted area of the bottom that remains immersed. At the time, Froude had conducted experiments to determine the friction resistance of planes of various lengths as a function of speed; however, our current friction coefficient dependence on Reynolds number had not yet been developed. In our current format, the horizontal component of frictional resistance is:

$$F cos\tau = C_F \frac{1}{2} \rho V^2 A cos\tau$$

where
C_F = friction coefficient of plane
F = friction force tangential to plane
A = wetted area of plane (m^2)
V = velocity of fluid with respect to the bottom (m/s) (for simplicity, V is taken as the free stream velocity. More information on the subject is provided by Savitsky, 1964)
ρ = mass density of water (kg/ m^3)

Froude's equation of $F_{pounds} = 0.01 A_{ft^2} V^2{}_{knots}$ (Provided in Appendix C) corresponds to a friction coefficient of around 0.0035, which is very typical for model-scale friction coefficients.

Thus, adding the pressure drag and the frictional drag, the total resistance of the planing hull is:

$$R = W \tan \tau + C_F \frac{1}{2} \rho V^2 A cos\tau$$

5.2 VERTICAL LIFT AND EQUILIBRIUM

It is necessary to know the wetted area in order to estimate the frictional resistance. The vertical component of lift generated by the hull must be equal to the weight of the boat when it is in equilibrium. Wetted area is typically considered a function of the weight the plate supports, the speed, aspect ratio and trim angle. Today, wetted area can be estimated by using one of the many flat-plate lift equations, such as the ones presented by Savitsky [11] or Shuford [12]. These types of

equations were not available until many years after Froude's death.

In order to estimate the lift of the planing surface, Froude started with an approximation for the lift on a fully-submerged flat plate (similar to a rudder). The equation was:

$$L_{pounds} = 5.7A_{ft^2}V^2{}_{knots}\sin\tau$$

Because only one side of planing surfaces are wetted (the bottom), Froude assumed that the lift should be reduced somewhat, and suggested:

$$L_{pounds} = 3A_{ft^2}V^2{}_{knots}\sin\tau$$

These hydrodynamic lift equations do not have a dependence on aspect ratio, which is a critical factor in the lift of hydrofoils and planing surfaces. The theoretical development at the time was limited to 2-D problems (with no aspect-ratio considerations) and there was very little test data available. In fact, it was nearly 60 years later when theory of planing craft became an area of serious study, with the most prominent early work by von Karman in 1929 [17] and Wagner in 1932 [18].

Froude's lift equation can be re-written as:

$$L = \frac{1}{2}\rho V^2 A C_L$$

where

$$C_L = 1.05\, sin\tau$$

which, at small angles, is equal to:

$$C_L \approx 1.05\,\tau_{rad}$$

(subscript *rad* provided to denote radians)

It is interesting to contrast this lift coefficient with the high-speed flat-plate lift term of the well-known Korvin-Kroukovsky equation [11], which converts to:

$$C_L = 1.03\sqrt{\frac{B}{L_M}}\tau_{rad}^{1.1}$$

From inspection it appears that Froude's lift coefficient applies for planing hulls with aspect ratios near unity.

5.3 TOTAL RESISTANCE

The total resistance may now be estimated. Solving the lift equation for area yields:

$$A = \frac{L}{\frac{1}{2}\rho V^2 C_L}$$

Substituting this area into the equation for total resistance, and recalling that, in order to be in equilibrium, neglecting the vertical components of frictional resistance and the propeller thrust, the lift must equal the weight supported ($L \approx W$):

$$R \approx W\tan\tau + C_F\frac{1}{2}\rho V^2\frac{W}{\frac{1}{2}\rho V^2 C_L}cos\tau$$

Collecting terms:

$$R \approx W\left(\tan\tau + \frac{C_F}{C_L}cos\tau\right)$$

Assuming that at small angles, $\tan\tau \approx \sin\tau \approx \tau_{rad}$ and $\cos\tau \approx 1$

$$R \approx W\left(\tau_{\mathrm{rad}} + \frac{C_F}{C_L}\right)$$

Using Froude's values of $C_F = 0.0035$ and $C_L \approx 1.05\,\tau_{RAD}$

$$R \approx W\left(\tau_{RAD} + \frac{1}{300\tau_{RAD}}\right)$$

Froude showed that this resistance is a minimum at a trim angle of 3.3 degrees and the minimum resistance $R_{MIN} = 0.12W$. The analysis confirms that, at high speeds, the resistance is not eliminated, but instead approaches a minimum value of $0.12W$. These values of minimum resistance and trim may be compared with Figure 16 of Savitsky [11], which shows that for a flat plate of 5-foot (1.5m) beam, the minimum resistance-to-weight ratio R/W = 0.12 at 4 degree trim. Although the results of Savitsky's study are dependent on beam, they demonstrate that Froude's finding from 90 years earlier was relatively accurate.

5.4 LIMITS OF APPLICABILITY OF FROUDE'S PLANING HULL RESISTANCE METHOD

Froude's estimate for optimum trim and minimum resistance is applicable with the following limitations:

a. Model Scale Reynolds Numbers
The friction coefficient of 0.0035 corresponds to a model, whereas full-scale friction coefficients are usually closer to 0.002, depending on length and speed, using the ITTC 1957 line. The optimum trim angle of full-scale vessels will be lower than this prediction because the friction term at full scale will be smaller in magnitude.

b. High Volume Froude Numbers
Savitsky [11] showed that, at low speeds, there is a substantial component of hydrostatic lift, which increases the lift per unit area of the planing surface. This is why the flat-plate resistance curve shown in Figure 15 has a resistance-to-weight ratio of less than 0.12 at many of the lower speeds.

The reader may wonder why R/W for the flat plate in Figure 15 is greater than 0.12 at the highest speeds. This is because that hull is operating at a sub-optimum trim.

If the model were fixed in trim, we would expect R/W to approach the value of 0.12

c. Aspect Ratio near Unity
Froude's lift equation is only good for an aspect ratio near unity because there was insufficient understanding of the effect of aspect ratio on lift in 1872. At the present time, the author has not been able to locate flat-plate test data from Froude's era to determine which aspect ratio he may have had data for.

d. Fixed Trim
The derivation provided by Froude is for the fixed-trim case, such as the sketch shown in Figure 23. It was not for another 90 years that Savitsky (1964) applied equilibrium of pitching moments to solve for trim of planing craft.

5.5 COMPARISON WITH DATA

A very brief (and far from exhaustive) survey of flat plate data on hand was made to evaluate the accuracy of Froude's results (within the range of applicability previously stated). Table 5 was prepared by identifying test runs by Savitsky and Neidinger [19] and Sottorf [20] that had trim angles near 3.3 degrees and aspect-ratio near unity. The models had beams of 200 – 300 mm, similar in magnitude to the Ramus models. The average resistance-to-weight ratio was 0.13, very close to Froude's estimate of 0.12.

Table 5: Flat Plate Model R/W

τ (deg)	L_M/Beam	F_∇	R/W	Source
4.0	1.2	3.6	0.13	Ref [19]
4.0	1.3	4.4	0.15	Ref [19]
4.1	1.1	3.2	0.11	Ref [20]
4.2	1.6	2.9	0.14	Ref [20]
3.7	0.7	5.1	0.13	Ref [20]

6. CONCLUSIONS

A high-speed stepped planing hull and three-point hydroplane concept were tested by William Froude in 1872, over thirty years before the lightweight internal combustion engine made planing boats possible. These test data were found in the UK House of Commons Hansard and re-analysed in a modern format. The analysis has shown that the Ramus hull suffered from a large increase in resistance due to the rounded stern; however, the vessel did plane, with the centre of gravity raised above the zero speed position. At the very highest speeds tested, the resistance begins to approach that of a Series 62 planing hull form with similar hull parameters.

New experiments were conducted to verify that the rounded-stern of the Ramus design was the cause of the large increase in resistance. These tests showed that the rounded-stern variant developed large downward forces, whereas the square-stern variant developed upwards forces. The striking difference seen between these tests of hulls that are virtually identical, with the exception of the rounded stern show that this was the likely cause of the problems with the Ramus design.

Froude's semi-empirical solution for the resistance of flat-bottomed planing craft was studied. This derivation was originally developed to combat the claims that Ramus-type hulls would rise out of the water and the resistance would approach zero at high speeds, The analysis is shown to be very similar to the procedures used by planing-boat designers today who rely on prismatic planing-hull prediction methods. Froude's methodology agrees with our current practice and includes most of the important variables, with the exception of aspect ratio (which was not understood at the time), hydrostatic buoyancy (which Froude omitted for clarity), and equilibrium of pitching moments. These effects were not combined into a single prediction technique until nearly 90 years later, with the work of Savitsky [11].

The original Ramus concept of a hull that would rise out of the water with the use of dynamic lift was not seen as useful at the original time of investigation. However, approximately thirty-five years after the Ramus tests, high-speed planing craft, powered by lightweight internal combustion engines, began to appear. Today, planing craft can be seen on most of the world's waterways, providing a satisfactory design solution for many commercial, recreational and military applications.

7. ACKNOWLEDGEMENTS

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APPENDIX A: FROUDE'S 1872 TEST DATA OF RAMUS HULL

FROUDE'S LARGE RAMUS HULL TEST DATA

Beam	423	mm
LOA	3048	mm
LWL	2879	mm
Displ.	53.1	kg
LCG	1300	mm fwd. transom (approx)
	20 C Fresh Water (assumed)	

F_∇	C_V	$R_{T,M}/W$	Heave (beams)	Trim (degrees)	L_M/L_{WL}	Re	R_R/W
0.13	0.13	0.000	-0.003	0.00	1.00	7.75E+05	0.000
0.27	0.25	0.002	0.001	0.02	1.00	1.55E+06	0.000
0.40	0.37	0.005	-0.004	0.04	1.00	2.31E+06	0.002
0.53	0.50	0.010	-0.005	0.06	1.00	3.08E+06	0.006
0.66	0.62	0.019	-0.012	0.06	1.00	3.84E+06	0.012
0.79	0.75	0.032	-0.015	0.17	1.00	4.61E+06	0.022
0.92	0.87	0.049	-0.022	0.24	1.00	5.37E+06	0.036
1.06	1.00	0.067	-0.038	0.35	1.00	6.14E+06	0.052
1.19	1.12	0.084	-0.050	0.54	1.00	6.91E+06	0.064
1.32	1.24	0.095	-0.048	1.09	1.00	7.67E+06	0.071
1.46	1.37	0.106	-0.047	1.79	1.00	8.46E+06	0.077
1.58	1.49	0.115	-0.045	2.20	1.00	9.20E+06	0.082
1.72	1.62	0.125	-0.041	2.48	1.00	9.99E+06	0.086
1.85	1.74	0.135	-0.036	2.70	0.99	1.06E+07	0.091
1.98	1.87	0.145	-0.016	2.88	0.93	1.07E+07	0.098
2.11	1.99	0.156	0.007	3.10	0.87	1.07E+07	0.106
2.25	2.12	0.168	0.028	3.34	0.82	1.07E+07	0.115
2.38	2.24	0.179	-	-	<u>0.76</u>	1.05E+07	<u>0.124</u>
2.51	2.37	0.191	-	-	<u>0.69</u>	1.01E+07	<u>0.135</u>
2.59	2.44	0.199	-	-	<u>0.65</u>	9.79E+06	<u>0.142</u>

Note: Underlined values of mean wetted length and residuary resistance are for speeds in which trim and heave data were not provided. For these speeds the mean wetted length was estimated for determining frictional resistance.

FROUDE'S SMALL RAMUS HULL TEST DATA – NORMAL RUNNING CONDITION

Beam	141	mm
LOA	1016	mm
LWL	960	mm
Displ.	1.97	kg
LCG	430	mm fwd. transom (approx)
	20 C Fresh Water (assumed)	

F_∇	Cv	$R_{T,M}/W$	Heave (beams)	Trim (degrees)	L_M/L_{WL}	Re	R_R/W
0.23	0.22	0.003	0.000	0.00	1.00	2.58E+05	0.001
0.45	0.43	0.009	-0.002	0.06	1.00	5.08E+05	0.004
0.69	0.65	0.025	-0.008	0.12	1.00	7.68E+05	0.015
0.91	0.86	0.052	-0.008	0.14	1.00	1.02E+06	0.035
1.15	1.08	0.090	-0.038	0.43	1.00	1.28E+06	0.065
1.37	1.29	0.117	-0.047	1.40	1.00	1.53E+06	0.082
1.61	1.51	0.142	-0.037	2.35	1.00	1.79E+06	0.096
1.83	1.72	0.171	-0.019	2.80	0.95	1.94E+06	0.116
2.06	1.94	0.202	0.001	3.28	0.86	1.98E+06	0.138
2.29	2.16	0.230	0.033	3.96	0.76	1.95E+06	0.160
2.52	2.37	0.254	0.092	4.76	0.67	1.87E+06	0.179
2.75	2.59	0.270	0.167	5.63	0.58	1.79E+06	0.191
2.98	2.81	0.281	-	-	<u>0.54</u>	1.80E+06	<u>0.196</u>
3.21	3.02	0.288	-	-	<u>0.50</u>	1.79E+06	<u>0.196</u>
3.44	3.24	0.293	-	-	<u>0.47</u>	1.80E+06	<u>0.194</u>
3.66	3.45	0.295	-	-	<u>0.45</u>	1.84E+06	<u>0.187</u>
3.89	3.66	0.298	-	-	<u>0.43</u>	1.87E+06	<u>0.182</u>
4.12	3.88	0.299	-	-	<u>0.42</u>	1.93E+06	<u>0.173</u>
4.35	4.10	0.300	-	-	<u>0.41</u>	1.99E+06	<u>0.164</u>
4.58	4.31	0.300	-	-	<u>0.40</u>	2.05E+06	<u>0.154</u>
4.70	4.42	0.300	-	-	<u>0.40</u>	2.10E+06	<u>0.147</u>

Note: Underlined values of mean wetted length and residuary resistance are for speeds in which trim and heave data were not provided. For these speeds the mean wetted length was estimated for determining frictional resistance.

FROUDE'S SMALL RAMUS HULL TEST DATA – UNSTABLE BOW-UP TRIM CONDITION*

LCG 430 mm fwd. transom (approx.)

F_∇	Cv	$R_{T,M}/W$	Heave (beams)	Trim (degrees)	L_M/L_{WL}	Re	R_R/W
2.74	2.59	0.271	-	-	-	-	-
2.98	2.81	0.285	-	-	-	-	-
3.21	3.02	0.295	-	-	-	-	-
3.44	3.24	0.303	-	-	-	-	-
3.66	3.45	0.310	-	-	-	-	-
3.89	3.67	0.316	-	-	-	-	-
4.12	3.88	0.321	-	-	-	-	-
4.35	4.10	0.327	-	-	-	-	-
4.58	4.31	0.333	-	-	-	-	-
4.81	4.54	0.340	-	-	-	-	-
5.04	4.75	0.348	-	-	-	-	-
5.27	4.97	0.358	-	-	-	-	-
5.49	5.17	0.370	-	-	-	-	-

FROUDE'S SMALL RAMUS HULL TEST DATA – FORWARD LCG POSITION*

LCG 615 mm fwd. transom (approx.)

F_∇	Cv	$R_{T,M}/W$	Heave (beams)	Trim (degrees)	L_M/L_{WL}	Re	R_R/W
2.63	2.48	0.287	-	-	-	-	-
2.74	2.58	0.294	-	-	-	-	-
2.97	2.80	0.310	-	-	-	-	-
3.20	3.02	0.322	-	-	-	-	-
3.44	3.24	0.334	-	-	-	-	-
3.65	3.44	0.344	-	-	-	-	-
3.90	3.67	0.355	-	-	-	-	-
4.12	3.88	0.364	-	-	-	-	-
4.35	4.10	0.374	-	-	-	-	-
4.54	4.28	0.382	-	-	-	-	-

*Note: Froude did not report Heave or Trim for the Forward LCG position or for the unstable bow-up trim condition, making accurate determination of wetted area and residuary resistance impossible.

APPENDIX B: 2014 ROUNDED AND SQUARE-STERN TEST DATA

RAMUS STERN
Beam = 140 mm
λ = 3.4 beams (fixed heave)
τ = 6 degrees

C_V	Normal (N)	Tangential (N)	λ (beams)
0.00	10.83	-1.03	3.3
0.26	10.72	-0.92	3.4
0.51	10.33	-0.63	3.4
0.78	9.22	-0.20	3.5
0.91	8.29	0.14	3.5
1.04	7.82	0.43	3.5
1.17	7.76	0.61	3.5
1.30	7.71	0.82	3.5
1.56	8.00	1.12	3.4
1.82	7.49	1.65	3.4
2.08	7.23	1.85	3.4
2.34	6.77	2.30	3.4*
2.61	5.98	2.54	3.4*
2.87	4.89	3.18	3.4*
3.14	3.72	3.66	3.4*

SQUARE STERN
Beam = 140 mm
λ = 3.4 beams (fixed heave)
τ = 6 degrees

C_V	Normal (N)	Tangential (N)	λ (beams)
0.00	12.17	-1.09	3.3
0.26	12.29	-0.98	3.3
0.52	12.35	-0.72	3.4
0.78	12.00	0.00	3.4
0.91	11.74	0.27	3.5
1.04	11.74	0.46	3.4
1.17	12.39	0.56	3.4
1.30	13.35	0.66	3.5
1.56	14.98	0.84	3.4
1.82	16.83	1.11	3.5
2.08	18.64	1.34	3.4
2.34	20.64	1.42	3.4*
2.60	22.83	1.79	3.4*
2.86	25.24	1.90	3.4*
3.12	28.17	1.91	3.4*

* Underwater photographs were not available at highest towing speeds; however wetted length remained constant at planing speeds for these fixed trim tests.

APPENDIX C: FROUDE'S DERIVATION OF MINIMUM RESISTANCE OF PLANING CRAFT

The following is reproduced from Froude's report to the Admiralty on the Ramus ship concept (Reference 1 and 8). It is reprinted here because it is of general interest to boat designers and researchers alike, and because the original references are difficult to locate.

"The entire resistance at extreme speeds may be regarded as reduced to two elements. (1) The horizontal component of the normal pressure on the inclined plane. (2) The surface friction of so much surface as remains immersed. The former is evidently identical with the horizontal component of the pressure on the inclined plane due to the weight supported; and since, at the extreme speeds we are considering, the whole ship or float may be assumed to be lifted, the force in question is the weight of the ship multiplied by the inclination of the plane. To determine the latter we must know (a) how much surface is in contact with the water, and (b) what is the law of frictional resistance in terms of surface of contact and of speed.

a. *The surface in contact must clearly be that which, taking account of the oblique pressure [force] supplied dynamically by the water, and of the speed, will yield an upward pressure [force] equal to the entire weight of the ship, and some explanation is necessary in reference to this point.*

The question may be stated as follows [See Figure 27]: Assuming the area (A) to be moving through a fluid along line B C, which forms an angle (ϴ) with the plane, and that speed is (V), what is the total normal pressure [force] (P) in terms of A, V and ϴ?

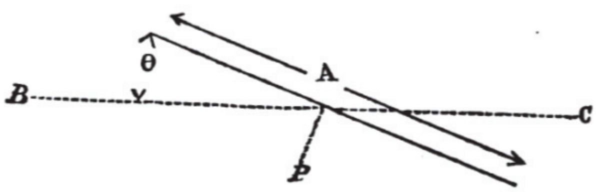

Figure 27: Planing on a Flat Free Surface

There can be little doubt that P is approximately directly as (A) and directly as (V^2), and it used to be held that it was as $sin^2\theta$. But it is now well known to those who have had occasion to verify this latter condition that instead of being as $sin^2\theta$ it is much more nearly as $sin\theta$ simply. A careful study of the whole question, and a careful analysis of all the best experiments on record, had long since satisfied me that up to ϴ=10-degrees it is very close to the truth to say,

$$P \ = \ 4.8\, A\, V^2 \,sin\, \theta$$

where,

P is the normal force in pounds
A the area in square feet and
V the speed in knots.

But this expression represents the force which acts on a plane wholly immersed, and therefore subject, not only to a positive pressure on its anterior face, but also a negative pressure on its posterior face. Whereas the case in question is one in which the plane is moving along the surface of the water, and can therefore only experience the positive pressure on the lower face, which is that towards the water, and in view of this condition it seems safe to put a lower coefficient, and say

$$P \ = \ 3\, A\, V^2 \,sin\, \theta$$

or to shape this into the form in which it is required so as to give A in terms of the other conditions.

$$A \ = \frac{P}{3\, V^2 \,sin\, \theta}$$

b. *The experiments which I have made here on surface friction, and which I have already reported, show that it will be sufficiently near the mark to treat the resistance due to it as proportional to the square of the speed, and if a very short area be taken account of (as indeed the circumstances require), it is practically correct to take*

$$F \ = \ 0.01\, A\, V^2$$

where,

F is the total frictional resistance in pounds
V the speed in knots, of the surface in its own plane.
A is the wetted area in square feet.

And it may be observed, that in the condition of "lift" finally attained the area exposed to frictional action will be simply the residuary under-surface of the inclined plane which supplies the support, since the speed of transit will entirely denude the sides and stern of contact with the water.

The conditions which have been traced out may now be brought together into regular mathematical form.

R = total resistance (in pounds)
W = total weight lifted (in pounds)
P = normal pressure [force] on plane (in pounds)
F' = Frictional resistance on plane (in pounds)
F = Horizontal component of F' (in pounds)
A = Area of surface in contact with water and supplying support by P (in square feet)
V = speed (in knots)
V' = speed along inclined plane (in knots)
ϴ = inclination of plane (radians)

Then

$$R = Wtan\theta + F$$

Now

$$F = F'cos\tau$$

and

$$F' = 0.01AV'^2 = 0.01AV^2cos^2\theta$$

Since

$$V' = V\,cos\theta$$

therefore

$$F = 0.01AV^2cos^3\theta$$

But in order that A may be such as to supply the due support

$A = \frac{P}{3V^2 sin\theta}$ *and* $P = Wsec\theta$

Therefore,

$$A = \frac{Wsec\theta}{3V^2 sin\theta}$$

Thus

$$F = 0.01\frac{Wsec\theta}{3V^2 sin\theta}V^2cos^3\theta = \frac{1}{300}Wcot\theta cos\theta$$

And

$$R = W\left(tan\theta + \frac{1}{300}cot\theta cos\theta\right)$$

For the small angles with which the investigation is concerned, we may take $tan\theta = \theta$ *and* $cos\theta = 1$, *and with this simplification the equation becomes*

$$R = W\left(\theta + \frac{1}{300\theta}\right)$$

The form of this is such that, by either greatly increasing or greatly diminishing Θ, R will be increased, so that the idea of obliterating resistance by making the inclined plane indefinitely flat is clearly illusory; and again, it follows that Θ may be so selected as to reduce R to its lowest possible amount, which, as will be seen, is by no means small.

Differentiating the equation we have –

$$\frac{dR}{d\theta} = W\left(\theta + \frac{1}{300\theta^2}\right)$$

And if $\frac{dR}{d\theta} = 0$*so as to give R its minimum value*

$\theta^2 = \frac{1}{300}$ *or* $\theta = \frac{1}{17.3}$ *[or 3.31 degrees]*

That is to say, the inclination which will give the minimum final resistance is about 1 in 17; and the minimum resistance about 2/17ths of the weight of the ship.

The rationale of the demonstration is easily intelligible. It is simply this:

If when the ship has become wholly lifted on the inclined plane we seek to diminish that element of resistance which consists of the horizontal component of the weight supported on the incline, by reducing, say halving, the steepness of the plane, then in order with this flattened inclination the dynamic action of the water should yield the same support, the plane must assume a doubled area of immersion, and this doubled area will involve a double frictional resistance.

But if we seek again to diminish the area of immersion by increasing the speed, the friction per square foot will be increased in the same ratio as the lifting force per square foot, and the immersed surface, though reduced in area, will retain the doubled frictional resistance which the halved steepness of the inclination introduced. Thus while we reduce one element of the resistance in any given ratio, we at the same time increase the other element in precisely the same ratio, and their combined amount cannot be reduced below the limit at which it stand when the two elements are equal.

It would not have been difficult to make the investigation somewhat more general, so as to include the growth of the resistance during at least the latter part of the scale of speed, before the lifting is virtually complete; but this would have added to its length, and diminished its clearness; as it stands, it sufficiently deals with that portion of the question, in which, because of its obscurity gave room for imagination, the hopes of a fascinating theory are tempted to make a final stand.”

William Froude, 1872